Title: **WEIGHTED LIKELIHOOD NEGATIVE BINOMIAL REGRESSION**

The talk presents a robust and efficient method for parameter estimation in negative binomial regression. The proposed method is based on the weighted likelihood approach (Agostinelli and Markatou, 1998), which has been showed to provide high breakdown point and fully efficient estimates in regression frameworks with i.i.d. errors. This approach uses a measure of disparity between the empirical distribution of the residuals and the theoretical error distribution to build weights. Therefore, when the errors are not i.i.d., as in the negative binomial regression case, its application is more difficult.

The proposed method uses "tail probabilities" (see below), which are i.i.d. even when the errors are not, to build the weights.

Let $NB(\alpha,\mu)$ be the family of negative binomial distributions and let $Y_{\alpha,\mu} \sim NB(\alpha,\mu)$. Then $E(Y_{\alpha,\mu}) = \mu$ and $Var(Y_{\alpha,\mu}) = \mu + \alpha\mu^2$. Consider a negative binomial regression model with response $Y_{\alpha_0,\mu_0(x)} \sim NB(\alpha_0, \mu_0(x))$, where x is a covariate vector and $\mu_0(x) = h^{-1}(\beta_0^T x)$, with h a given link function.

Let $(x_1,y_1),\dots,(x_n,y_n)$ be a random sample and $u_1,\dots,u_n$ be random numbers generated from the uniform distribution on [0,1]. Then the "tail probabilities"

$$p_i = P\left(Y_{\alpha_0,\mu_0(x_i)} \le y_i\right) - u_i P\left(Y_{\alpha_0,\mu_0(x_i)} = y_i\right), \qquad i = 1,\dots,n$$

form a sample from a uniform distribution on [0,1]. Let $q_i = \Phi^{-1}(p_i)$, where Φ is the standard normal cdf. Then $q_1,\dots,q_n$ is a sample from a standard normal distribution. A set of weights is derived from a measure of disparity between the empirical cdf of the q_i and the standard normal cdf. These weights are used in a weighted likelihood procedure which yields robust and fully efficient estimates of α_0 and β_0.

The algorithm used to compute these estimates needs to be given a robust and $\sqrt{n}$–consistent starting point. To this purpose we use a combination of Han's maximum rank correlation estimator (1986) and an M-type estimator.

The proposed method is actually quite general and can be applied to a large variety of regression frameworks where the errors are not necessarily i.i.d. and can involve shape parameters (like α in the negative binomial regression case).

References:

Agostinelli, C. and Markatou, M. (1998), "A one-step robust estimator for regression based on the weighted likelihood reweighting scheme", *Statistics & Probability Letters*, 37, 341–350

Han, A. K. (1987), "Non-parametric analysis of a generalized regression model", *Journal of Econometrics*, 35, 303-316